(Developing) Teacher Discourse Moves: A Framework for Professional Development

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We describe our ongoing efforts to design materials for supporting secondary mathematics teachers in using a set of Teacher Discourse Moves purposefully in order to develop classroom discourse that is both productive and powerful for students’ learning. We focus on secondary mathematics classroom discourse because mathematical language and meanings get increasingly complex beginning in middle school, and most discourse-related work in mathematics education has focused on elementary school classrooms. We make explicit both the concepts we use and the translation of these theoretical concepts into ideas useful for practice. This article contributes to ongoing discussions about making visible the work of developing research-based professional development materials.

Key words: Classroom discourse; Professional development; Secondary teaching

When students learn to engage in mathematical argumentation and conceptual explanations in increasingly complex ways, they improve their learning (Chapin & O’Connor, 2004; Chapin, O’Connor, & Anderson, 2009). To do this, teachers’ need to establish classroom norms and culture with meaningful discourse at the center (e.g., Cobb, Boufi, McClain, & Whitenack, 1997; Hufferd-Ackles, Fuson, & Sherin, 2004; Stein & Lane, 1996). Mathematics teachers’ discourse patterns, however, remain primarily in an Initiate-Respond-Evaluate (I-R-E; Mehan, 1979) format (e.g., Stigler & Hiebert, 1999), even when teachers attempt to change their classroom practices (Herbel-Eisenmann, Lubienski, & Id-Deen, 2006).

In response to this evidence, we have been designing and piloting professional development (PD) materials. The long-term goal of the project is to support secondary mathematics teachers in becoming purposeful about engaging students in mathematical explanations, argumentation, and justification. We have chosen to focus on discourse at the secondary level because mathematical language and meanings become increasingly complex beginning in middle school, and most discourse-related work in mathematics education has focused on elementary school classrooms.

As part of this work, we have been building on ideas from Chapin, O’Connor, and Anderson (2003), who focused on mathematics discourse in upper elementary school classrooms. Our materials modify their “talk moves” to make more central a) the context of secondary school mathematics classrooms and the ways in which mathematical discourse gets more sophisticated as students progress through their mathematical experiences; and b) students’ opportunities to learn mathematics, by attending broadly to students’ experiences and learning. Following Gresalfi and Cobb (2006), we define “opportunities to learn” as including not only students’ access to mathematical content and discourse practices but also their access to (positional) identities as knowers and doers of mathematics. (as summarized in Esmonde, 2009, p. 250)

Using a modified set of talk moves as a centerpiece, the PD materials support participants in becoming purposeful about developing productive and powerful classroom discourse. As participants learn about, contemplate, and intentionally plan for classroom discourse, they become more purposeful about how they enact classroom discourse practices (Herbel-Eisenmann & Cirillo, 2009) that are both productive and powerful for student learning. By productive we mean the ways in which the participant’s
Instructional practices can support students’ “access to mathematical content and discourse practices.” By powerful we mean the ways in which the teacher’s discourse practices can support students’ “(positional) identities as knowers and doers of mathematics.”

In the remainder of this manuscript we describe the architecture of these PD materials, describe some of the ways in which we have incorporated the ideas of productive and powerful discourse into the PD materials, and share some of what we have seen from our first pilot conducted during the 2011–2012 school year.

**Overview of the Structure of the Materials**

The Mathematics Discourse in Secondary Classrooms (MDISC) professional development materials are practice-based materials. They meet the criteria for practice-based materials in two important ways. First, the activities for professional development represent a range of work in which teachers engage; they are based on various artifacts of practice, drawn mostly from a large database of classroom videos of secondary mathematics teachers who reflected on their discourse practices in a previous project. Specifically, the materials provide opportunities for participants to solve mathematical tasks, consider the planning and enactment of lessons, reflect on practice, and assess student learning. Second, these artifacts of teaching practice—tasks, lesson plans, narrative and video cases, student work—are positioned as objects of inquiry from which general principles about teaching and learning mathematics in a discourse-rich classroom can emerge.

The broader structure of the materials includes an introductory module, five constellations of activities, and a capstone experience. The introductory module serves the primary purpose of establishing a learning community that will support participants to work together on aspects of mathematics classroom discourse. The five constellations are described in Appendix A. After Constellation 5, the capstone activities prepare secondary mathematics teachers to engage in cycles of action research related to their discourse practices. In an ideal enactment of these materials, the introduction and constellations would be used for a yearlong study group consisting of about 40–50 hours of PD, and the capstone would lead into 2-year cycles of action research to support teachers as they work toward becoming more purposeful about aspects of classroom discourse in their own classrooms.

Each constellation of activities includes a “high cognitive demand” (Smith & Stein, 1998) task, a video or narrative case of a teacher enacting that task, other classroom artifacts (e.g., student written work, textbook excerpts) related to the task, and Connecting to Practice activities that ask participants to observe, record, and/or reflect on core discourse-related ideas in their classroom practice. Additionally, there are “touchstone” documents that articulate information and definitions of essential concepts in the materials.

**Talk Moves as a Beginning Framework**

Our beginning conceptualization for these practice-based materials was inspired by Project Challenge, a collaboration in Boston focused on low income, linguistically diverse urban schools. Project Challenge combined a well-planned curriculum for grades 4–7 with thoughtful incorporation of specific talk moves. The PIs documented how students’ ways of talking and reasoning in mathematics changed dramatically in both qualitative and quantitative comparisons (Chapin & O’Connor, 2004; Chapin et al., 2003). Because these talk moves were shown to be productive for student learning, we made them central to the MDISC materials. The specific talk moves included:

a. Revoicing
b. Asking students to restate someone else’s reasoning
c. Asking students to apply their own reasoning to someone else’s reasoning
d. Prompting students for further participation
e. Using wait time. (Chapin et al., 2003; see pp. 12–16)

**Rearticulating the Talk Moves as Teacher Discourse Moves**

Our initial year of development involved creating a constellation of activities for each of the talk moves. We began the work by watching videos from the large database to identify instances of the talk moves that contributed to the development of mathematical ideas, relating
to the notion of productive discourse. In project meetings and discussions with advisory board members, however, several important nuances related to the talk moves emerged. First, the talk moves depended on nonverbal aspects of communicating (i.e., gesture, written language, and information displays) that were important to mathematical sense making. Second, the talk moves did not explicitly attend to issues of authority and control that arise when teachers focus on their classroom discourse (Herbel-Eisenmann, Drake, & Cirillo, 2009; Herbel-Eisenmann & Wagner, 2010). Third, when the talk moves were used in combination (rather than individually), the impact on the quality and quantity of student contributions was quite substantial. Thus, we shifted focus from addressing each talk move individually and instead organized the materials based on what the talk moves together seemed to accomplish.

In revising the talk moves, we wanted to highlight the reality that in most classrooms, teachers have more power than their students to shape the classroom discourse. Because we were developing PD for teachers, we also wanted to highlight that these moves are actions participants can thoughtfully plan for and use to open up the classroom discourse. We decided to call the moves teacher discourse moves (TDMs) and use verbs to draw attention to the action participants might take. Finally, as we expanded our descriptions of each TDM, we split the adding on talk move into two TDMs, inviting student participation and probing a student’s thinking, because this move seemed to support different aspects of classroom discourse.

Below we describe the six TDMs in the MDISC materials:

- **Waiting.** Waiting (i.e., using “wait time”), to provide students with time to process teacher questions and think about their responses, is critical to productive and powerful discourse. Although teachers are probably aware of the benefits of waiting after asking a question, a lesser known form of wait time involves waiting after a student responds. When this second form of waiting occurs, students’ responses can become more complex (Rowe, 1986), and students may be more likely to respond to their peers contributions.

- **Revoicing.** Revoicing occurs when a teacher restates or rephrases a student’s contribution. More specifically, revoicing has been defined as “the reuttering of another person’s speech through repetition, expansion, rephrasing, and reporting” (Forman, McCormick, & Donato, 1998, p. 531). An essential ingredient of what we call full revoicing lies in the second part of the teacher’s contribution (O’Connor, 2009). Full revoicing occurs when the teacher checks back with the original speaker and offers an explicit opportunity for students to respond to questions such as “Did I get that right?”

- **Asking students to revoice.** This move is similar to the revoicing move described above except that students are asked to do the revoicing. It requires that students listen to each other and gives them opportunities to revoice ideas in their own words.

- **Probing a student’s thinking.** This move is about following up with an individual student’s solution, strategy, or question. The goal here is to have the student elaborate on his or her ideas. For example, a teacher might ask how or why, or invite a student to come to the front of the room to provide additional information, such as a diagram. Probing may stem from a teacher’s genuine desire to know more about the student’s thinking, or it could be used to make a student’s thinking explicit for the benefit of other students.

- **Creating opportunities to engage with another’s reasoning.** This move involves asking students to engage with another student’s idea. For example, a teacher might ask the class to use a particular student’s strategy to solve a similar problem or to agree or disagree

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4 We would like to acknowledge the critical role these members played in the improvement of the MDISC materials. The members include Ryota Matsuura, Valerie Mills, Randy Philipp, David Pimm, Mary Schleppegrell, Ed Silver, and Peg Smith.

5 We do not mean to say that students do not shape classroom interactions. Rather, we see norms as being mutually constituted but recognize that teachers have more institutional power, for example, by virtue of their right to organize the learning environment and evaluate and assign grades.

6 Although we recognize that O’Connor and Michaels would probably not use revoicing this way, we use it here because it is how the teachers with whom we have worked used it when they talked about revoicing in relationship to their classrooms. To them, it made more sense to use revoicing in both cases because both involve re-saying something another person said. Yet they still recognized that teachers are the ones who typically do full revoicing because they are the ones who normally check back with students.

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with a solution. Another form of this move might be to ask students to add on to or revise another student’s explanation or conjecture. Effective use of this discourse move could be enhanced by the prerequisite use of other discourse moves and works best when students are actively listening to each other.

We developed the MDISC materials to provide opportunities for participants to notice and investigate the TDMs as specific tools that can be used intentionally to support productive and powerful classroom discourse.

**TDMs in the MDISC Materials**

To facilitate participants’ noticing the TDMs, we introduce TDMs by providing a transcript of a teacher, Mr. North, using the I-R-E interaction pattern in the beginning of his lesson and contrasting it with a later part of the lesson in which Mr. North uses many TDMs. Pilot participants were very critical of the I-R-E interactions, pointing out that the students do not get to say much. Because we intentionally try to avoid this kind of evaluation of practice in the MDISC materials,7 the facilitator materials provide support for a discussion about when and why teachers might employ an I-R-E pattern. We know from previous research that I-R-E is one of the most prevalent discourse patterns in classrooms, so we want facilitators and participants to think about when and why teachers might use it, rather than draw the conclusion that discourse is inevitably “bad” when one uses I-R-E. Similarly, when participants examine the interactions during the later part of Mr. North’s lesson, where he incorporates many of the TDMs, we ask the participants to talk about when and why teachers might want to use these moves to open up the discourse. Pilot participants expressed concern about how much time it would take to have discussions that incorporate the TDMs but were also critical of how quickly the pace of I-R-E interactions moved and how the discourse seemed more closed. This is the type of nuanced analysis in which we hoped participants would engage.

After we discuss how the interaction patterns are different in each of the transcripts, we offer a touchstone document that names the TDMs, defines them, and illustrates some forms and functions for each TDM. We then ask participants to find examples of each of the TDMs in Mr. North’s transcripts and to talk about what they see happening around the TDMs. Following this activity, we consistently ask participants to identify places where the teachers in the subsequent transcripts use these TDMs and the ways in which their use seemed to shape both the mathematical and the social aspects of the interactions. Also, in the follow-up Connecting to Practice assignment, we ask participants to audio- or video-record a lesson to see when and how they were using I-R-E and the TDMs. In Constellation 3, we consider how teachers can plan to intentionally use the TDMs toward achieving particular mathematical and social goals. In Constellations 4 and 5, we examine the use of the TDMs during the launching of a task, in participants’ interactions with small groups of students, and in the whole-class conclusion phase of work on a task.

**Interpreting the TDMs With Consistent Lenses**

As we were developing the MDISC materials and writing discussion questions that pushed participants to interpret what was happening in the cases, we found that we were missing interpretive lenses for discussing what was happening around the TDMs. We returned to our commitment to focusing on students’ opportunities to learn to guide our decisions about these interpretive lenses and considered: (a) the ways in which the TDMs can make mathematics classroom discourse productive for students; and (b) how to support students in seeing themselves as knowers and doers of mathematics, related to the idea of powerful mathematics classroom discourse. We turned to lenses and concepts that could support participants in interpreting the TDMs in relationship to these two aspects of opportunities to learn.

**Consistent Lenses for Interpreting the TDMs**

Our commitment to equitable practices (see Gutierrez, 2011) made it imperative to choose a theory of language that does not frame students’ learning in deficit ways (e.g., some theories assume that people are unable to learn certain kinds of language if they have not done so by a particular point in time; Gibbons, 2006). The theory that we chose—systemic functional linguistics (SFL; Halliday & Hasan, 1989; Halliday & Matthiessen, 2004)—addressed our challenges.

The relationship between context and language use is paramount in SFL; SFL assumes that language learning is intimately related to the cultural and situational context in which the learning takes place. The lexical (words or vocabulary) and grammatical (construction of words in meaningful ways) features of a context comprise a register (Halliday & Hasan, 1989). Three aspects of context

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7 We would like to thank Nanette Seago for helping us articulate to participants the following three reasons that the materials do not attempt to evaluate the examples of practice: (1) there is not enough data to make evaluations because participants are only seeing snapshots of teaching, not entire lessons or longer segments of practice; (2) the videos are not exemplars to follow, but examples of practice to explore; and (3) the videos are for investigation of teaching and learning, not for doing an evaluation of the teacher.
influence a register: what is being talked about (or the “ideational” metafunction of language); who is involved in the interaction and what their relationships are (or the “interpersonal” metafunction of language); and whether the mode of communication is written or spoken (or the “textual” metafunction of language). What is being talked about and whether it is being communicated verbally or in writing are important to developing classroom discourse that is productive; who is involved and their relationships relate to developing classroom discourse that is powerful. SFL assumes that if students do not learn to use particular kinds of language with meaning, it is because they have had too few opportunities to use that language meaningfully in relevant contexts. We describe these ideas in a touchstone document in the introduction to the MDISC materials because we want to be transparent and because we know it is common for people to use discourse and talk as if they are synonyms. We did not want participants entering the PD experiences with this more narrow definition of the term.

In order to support participants’ use of TDMs in ways that are purposeful, we adopted the Language Spectrum and positioning as lenses to analyze students’ opportunities to learn. That is, when participants analyze instances of the TDMs in transcripts, we ask them to consider what they think is happening in terms of the Language Spectrum and positioning. These lenses foster analysis of how the discourse might be productive and powerful for student learning. We summarize these key ideas in Figure 1 and delineate them in the following sections.

**Articulating a Lens for Considering “Productive” Discourse**

**The Language Spectrum**

Mathematics teachers often have students work in different communication contexts (CCs) (e.g., small group, whole class reporting, student written solutions, and textbook excerpts) that shape the ways students communicate—an idea described in the Language Spectrum. In the same way that a visible light spectrum distributes colors of different frequency ranges, we use the term Language Spectrum to emphasize the range of ways people communicate (or the characteristics of the texts they produce) in relationship to the CC. The Language Spectrum helps participants consider CCs and how those CCs help students gain facility over time with discipline-based ways of communicating. It is important to note that in this range of texts, one way of communicating is not better than another, just as red light is not better than orange. Instead, the Language Spectrum illustrates how CC affects the kind of language that students use, and, by extension, illuminates how important it is to consistently provide CCs in which students can use mathematically complex language.

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8 In SFL literature, the term used for this idea is mode continuum. As suggested by the secondary teachers with whom we have worked, we purposefully changed the term to Language Spectrum to eliminate confusion with the mathematical meaning of mode as well as participants’ tendency to rank-order excerpts when asked to put something on a “continuum.”

9 In following with SFL, we use the word text to mean a stretch of spoken or written communication that is produced in a context.

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**Figure 1. Summary of Key Ideas Used to Augment the Teacher Discourse Moves**
We illustrate the Language Spectrum by considering how language use might change in each of the following CCs:

a. a small group of students work at their desks to try to figure why \( \frac{b^m}{b^n} = b^{m-n} \);

b. one student from that group is asked to report back to the whole class;

c. a student writes a solution to the task; and

d. a textbook author writes a formal explanation.

In Table 1 a mix of hypothetical transcripts, real student dialogue, and a textbook excerpt illustrate characteristics of typical texts produced in each of these CCs. The examples illustrate characteristics of the type of language we would expect people to use within a particular CC.

As Gibbons (2009) pointed out, “The four texts in the Language Spectrum, taken together, represent a speaking-to-writing continuum” that characterizes the ways in which “the less shared knowledge there is between speaker and listener (or writer and reader), the more explicit language must become” (p. 48). Each column in Table 1 features similar mathematical content and correct solutions to the task, but the grammatical choices are quite different: the language (and symbolism) becomes more mathematically technical, the feel becomes less personal, and the mode of communication becomes formal written academic language as one reads from left to right.

We describe the characteristics of the spoken CCs (left two columns) first. In CC 1, the typical text (Text 1) that is produced can be characterized as language of:

Table 1
Illustrative Texts Corresponding to Different Communication Contexts

<table>
<thead>
<tr>
<th>CC1: Working in a small group</th>
<th>CC2: Reporting out to the whole class</th>
<th>CC3: Student writes a solution</th>
<th>CC4: Written description in a mathematics textbook</th>
</tr>
</thead>
</table>

**Student 1:** OK, so I think you just take this away from this, and then you just have, like, something on the top. Like, here and here [points at examples], there isn’t anything left. They all just cancel out. I think that’s why the rule works. You can cross out the numbers under here [points to the denominator].

**Student 2:** Couldn’t you have, like, more on the bottom?

**Student 3:** Remember when we had that assignment where we had to write out what all the exponents meant, like three to the fifth power was three times itself five times? And when we did that with the division problems, you could cancel out the same amount on the top and bottom? Like, if there are five on top and three on the bottom, you can cancel three of them and just have two left. But we just did that problem with \( b \) to the \( m \) on top and \( b \) to the \( n \) on bottom. So, just like we said, five minus three is two, you do \( m \) minus \( n \) and that’s what you have left. That’s what we got.

When you divide exponents with the same base, like \( \frac{b^m}{b^n} \), there are \( m \) copies of \( b \) in the numerator and \( n \) copies of \( b \) in the denominator. You can simplify this expression because copies of \( b \) in the numerator will cancel with copies of \( b \) in the denominator. Since \( \frac{1}{b^n} = b^{-n} \), \( \frac{b^m}{b^n} = b^{m-n} \). When you multiply exponents with the same base, we add the exponents, so \( \frac{b^m}{b^n} = b^{m-n} \).

In the case of division where the bases of the exponential expressions that are divided are the same, such as \( \frac{b^m}{b^n} \), where \( b \), \( m \), and \( n \) are rational numbers, the result is \( b^{m-n} \). This is a consequence of the multiplication rule for exponents with like bases.

\( \frac{b^m}{b^n} = b^{m-n} \)
Interestingly enough, even when we have engaged in similar activities with mathematicians, they also seem not to have explicitly noticed the interaction. Because the problem is within reach, students are likely to point and use language that is context dependent\(^\text{10}\) (e.g., here and here, this, it). Students ask each other interactive and clarifying questions (e.g., Right?) and tell each other to do things. References to mathematical ideas may not be very precise (on the top, cross out the numbers, under here, bottom).

In CC 2, the associated Text 2 can be characterized as language of recounting experience. Because others in the room were not in the small group, students’ contributions are more specific (e.g., on the top), use more precise mathematical terms (e.g., \(b\) to the \(m\) minus \(n\), exponents), and provide logical connectors (e.g., because) to explain their reasoning. Yet the speaker might recognize some shared experiences with others in the room (e.g., Remember when we . . .). Sometimes the description provided is organized chronologically (e.g., and then) and is reported in the past tense because the events occurred previously. The human actors (e.g., we, I) who worked on the problem are recognized.

CCs 3 and 4, the two right columns in Table 1, are written texts. In CC 3, the associated Text 3 can be characterized as language of generalizing. In this case, the explanation needs to make sense to an audience that did not participate in the activity, so students provide text that explains what they did (e.g., When you divide exponents . . .). This type of framing shows that the writer understands that the audience may not know what he or she is describing, making the description less reliant on the actual context in which the activity took place. A more general actor—\(you\) (as in “one”)—may be used to generalize processes at a broader level than when a student describes what she did using the pronoun \(I\) or what her group did using the pronoun \(we\) (Rowland, 1992). More precise mathematical terms are used (e.g., numerator, expression), and specific mathematical processes are named (e.g., divide, simplify), making the text more mathematically dense than the previous two texts. Because the text is organized as a general explanation of the solution process, it is written in the timeless present tense and uses terms such as when to establish conditions and because and since to offer reasons and results.

In CC 4, the associated Text 4 includes many characteristics of the mathematics register (Halliday, 1978; Pimm, 1987). The text has no human actors; it is not about what people do or did but instead about relationships between things, the subjects are abstract entities such as the result and a consequence, and passive voice is used (e.g., expressions are divided). Related to this, the processes change from action verbs such as add to relational verbs such as are or is, and the relationships expressed indicate equivalence; that is, \(m\) and \(n\) are rational numbers, and the result is \(b^{m-n}\). Mathematical symbols replace words in order to create efficient and precise expressions of ideas. Conditions are stated as general laws and not as a result of “what you do.” Based on the mathematical community’s requirements for communicating, this text is the most mathematically sophisticated and precise.

Having teachers distinguish features of these texts and contexts is important because teachers evaluate students’ correct solutions differently when students use linguistic features more like the mathematics register, even though many teachers cannot explicitly articulate their criteria for this evaluation (Morgan, 1998). For example, the pilot teachers said things such as, “This text flows better than the others,” but until they were pushed to do so did not identify which grammatical features of the text actually achieved this purpose. They also did not recognize that their judgment of student work was informed by years of enculturation in mathematical practices that were implicit to them.\(^\text{11}\)

These are important aspects of communication to make explicit because teachers and other students also have been shown to treat students differently depending on whether or not they consistently use features of the mathematics register correctly (e.g., Esmonde, 2009). So, participants need to be aware of the particularities of the mathematics register and to openly support students’ use of it.

Most of the teachers with whom we have worked have focused almost exclusively on mathematics vocabulary and do not consciously notice many other characteristics of the mathematics register (see Schleppegrell, 2007). Yet meaning is developed when students learn to put vocabulary words together in mathematically meaningful ways (e.g., Herbel-Eisenmann & Otten, 2011). Precise use of discourse as described in the concept of the mathematics register (e.g., Halliday, 1978; Pimm, 1987; Schleppegrell, 2007), however, requires teachers also to understand, for example, how dense noun phrases, nominalizations,\(^\text{12}\) logical connectors, and

\(^{10}\) In fact, in some cases, if an observer were to turn her back on the group, it might not be apparent that students are talking about mathematics; there are few mathematical references in CC 1 because students can use context-dependent terms.

\(^{11}\) Interestingly enough, even when we have engaged in similar activities with mathematicians, they also seem not to have explicitly noticed the ways in which they use language to achieve mathematical purposes as described in the mathematics register.

\(^{12}\) Examples of nominalizations include: (a) “rotate the triangle 90 degrees in the plane” becomes “the rotation,” and (b) “taking the limit of a difference quotient” becomes “the derivative.” In some cases, the renaming of the process as a noun involves a word from the description of the process (like rotate becoming rotation), and in other cases, a new word is introduced to rename the process as a noun (as in (b), above). Nominalization removes human participants from the statement, giving agency to mathematical objects and processes rather than to the people who are doing the mathematics.
verbs\textsuperscript{13} are used to communicate about mathematics, as well as the ways in which many of these linguistic constructions can be challenging for students.

The Language Spectrum traces the development of mathematical complexity\textsuperscript{14} in the verbal and written language across these texts. Although we have numbered them as Texts 1, 2, 3, and 4, the use of such language is not as linear or demarcated as it appears. In fact, many of these forms of talking and writing should occur throughout the teaching-learning process, because each of these texts allows students to express their understandings and helps them to develop more formal ways of talking and writing about mathematics.

The Language Spectrum in the MDISC Materials

We introduce the Language Spectrum in Constellation 1 by having participants analyze two different sets of correct solutions to a mathematical task, paying attention to how students use language to communicate in each example. Then participants are asked to generalize about the particular ways of using language that they notice between the two sets of correct solutions. Pilot participants (and some attendees who have engaged in this activity during presentations we have given) have noticed many of the characteristics we describe in relationship to Table 1 above.

Typically, users\textsuperscript{15} notice precise mathematical vocabulary early in the discussions of the student work, and the use of vague language and gesturing appears next because users notice that not all of the solutions use a lot of precise mathematical vocabulary. The different ways of relying on diagrams and examples in the communication usually emerge fairly quickly, too. Most users notice the more formal logical connectors that appear in the written work but are less apparent in the verbal forms of solutions. Typically, users have had to be probed to attend to the role of human actors (\textit{I, we}) in comparison to mathematical actors (\textit{triangles, graphs}) and the shifts in the kinds of verbs that are used (from actions the people took toward relational verbs such as \textit{is}). After participants engage in this analysis and discussion of the sets of student solutions, we offer a touchstone document about the Language Spectrum for participants to read and discuss.

Because the mathematics register (Text 4 in the Language Spectrum) describes how the mathematical community uses language and meaning systems, we explore this idea further in the MDISC materials. Participants analyze textbook excerpts to identify characteristics of the mathematics register and read a touchstone document that further explains the idea. We intend for these experiences to encourage participants to consider how the mathematics register might provide a way of thinking long-term about how to assist students’ developing proficiency in communicating about mathematics.

Many pilot participants shared that they primarily attended to mathematics vocabulary when they were teaching and assessing students and needed to pay attention to some of these other ways of using language in mathematics. Because we anticipated that participants would focus more attention on mathematics vocabulary, we have them look at some textbook excerpts in which students are being directed to do or solve some tasks. We ask them to identify characteristics of the mathematics register and consider why mathematical texts might be difficult for students. Some pilot participants expressed dissatisfaction with the language requirements of the mathematics register and stated that textbook authors should not be communicating this way if they want students to be able to access the information. They wanted to know why students had to learn to use language in these ways. This objection led to a conversation about access and how, for students to be seen as legitimate participants in mathematics discourse, they need to be recognized as members of the community through the ways in which they communicate.

We incorporate aspects of the Language Spectrum in our questions about the cases and student work throughout the subsequent constellations. Because the Language Spectrum focuses on four different CCs, we include examples of each of the four CCs at various points across

\textsuperscript{13} In mathematics, in particular, the relational verbs \textit{be} and \textit{have} and other related verbs (\textit{means, equals, etc.}) are grammatically challenging for students. As an example, students may have only been implicitly shown that the word \textit{is} takes on two very distinct meanings in mathematics. The \textit{is} in a \textit{square} is a \textit{quadrilateral} is different than the \textit{is} in a \textit{prime number} is a \textit{number} that \textit{can only be divided by 1 and itself}. The former describes a classification of an object (a square is a subset of the set of quadrilaterals), whereas the latter describes an identity relationship (a prime number is equivalent to a number that can only be divided by 1 and itself). It is important to note that unlike the prime number example, the first statement is irreversible (i.e., a quadrilateral is not necessarily a square). Thus, students may need explicit discussions about these differences in the use of the word \textit{is}, or they may not understand the intended relationships. An important mathematical difference exists between \textit{is} as a classification versus \textit{is expressing an identity relationship.}

\textsuperscript{14} We emphasize \textit{mathematical} here because we recognize that the practices of the mathematical community determine “complexity” in this case. We have found communicating this aspect of the work to be particularly difficult because we do not want participants to think that language use outside of the mathematics register is not complex.

\textsuperscript{15} Here we refer to users of this particular activity because it includes pilot participants and people who have attended conference presentations we have done using this activity.
the constellations so participants can continue to make sense of the relationships between language use and the CCs. In the following section, we share specific examples of the connections between the Language Spectrum and TDMs that appear in the MDISC materials.

The Language Spectrum and the TDMs

The Language Spectrum is a tool for thinking about (a) how communication context shapes language; and (b) how students typically use language in particular communication contexts. For example, if some students use the kind of language we might expect them to use in a small group (i.e., context-dependent references like this or that) even when they are writing solutions, it may be that these students need additional support to use language in more precise ways when writing a solution (CC3). The TDMs can be particularly useful here. For example, a teacher might probe a student’s thinking during small-group work, asking him what this refers to and pointing out that when he writes his solution, he needs to name the mathematical objects or processes more precisely. Or a teacher may decide to revoice a student contribution, making it more mathematically precise, and then explain why she used this other language. She may also choose to ask a student to revoice, encouraging students to use more of the mathematical vocabulary that they know.

Alternatively, a teacher may decide that the students need to have more exposure to the other CCs. For example, if students only work through mathematics individually or in small groups, they may not have had enough opportunities to use language in ways that are less context-dependent by reporting out to the whole class. In order to provide support for students to explain their thinking, a teacher may choose to probe a student’s thinking. To support the transition between reporting out to writing a solution, a teacher may record a revoice on the board and ask students what changes should be made to these spoken words to make it a more mathematically precise written explanation for an absent audience. Finally, a teacher may anticipate that a particular textbook explanation will be difficult for students to understand and so choose to have someone read it aloud and then ask students to revoice the textbook in words that make more sense to them.

Teachers can purposefully scaffold students’ facility with the mathematics register over a period of time by considering interactions between the Language Spectrum, CCs, and the TDMs. The goal, then, is not to speed through having students use language in CCs 1, 2, and 3 to help them develop language similar to Text 4. Rather, as teachers and students work in different CCs, they should move back and forth between these ways of communicating to support student learning. Sometimes the formal written mode needs to be unpacked in less formal ways in order for students to make sense of the ideas. And sometimes a teacher needs to introduce formal mathematical language for ideas about which students talk. The TDMs play a paramount role in moving back and forth among the kinds of texts typically produced in these different CCs.

When teachers use the Language Spectrum to consider how they scaffold language learning over time, they can provide access to every student because the rules of the game are apparent to everyone (Gibbons, 2006, 2008). Yet many mathematics teachers and students operate on a transmission model of communication, which does not support students in this way. In fact, as students progress through the more typical mathematics pipeline (Gutierrez, 2011), fewer students emerge feeling empowered or are even interested in mathematics. How, then, might teachers consider the ways that classroom discourse practices can purposefully support students to see themselves as capable knowers and doers of mathematics? That is, although the Language Spectrum helps participants consider the productivity of their classroom discourse, it does not draw explicit attention to whether the discourse is powerful for student learning. Here the idea of positioning is useful.

Articulating a Lens for Considering “Powerful” Discourse

Teachers who are purposeful about their mathematics classroom discourse must also contemplate how classroom discourse can be powerful for students’ developing identities and dispositions (Boaler & Staples, 2008; Featherstone, Crespo, Jilk, Oslund, Parks, & Wood, 2011). In fact, there is a strong link between equity issues and classroom discourse (see, e.g., Herbel-Eisenmann, Chopin, Wagner, & Pimm, 2011). Attending to the interpersonal function of language is paramount to changing some of the beliefs and images that students have about mathematics and about themselves as mathematical learners. Thus, we turn to the idea of positioning to consider students’ developing identities and dispositions, both in relationship to people (student-to-student interactions; teacher-to-student interactions) and what students come to see as the activity of “doing mathematics” in classrooms and how they see themselves in relationship to this activity.

Positioning

In the MDISC materials, we define “positioning” (Harré & van Langenhove, 1999) as
the ways in which people use action and speech to arrange social structures. . . . This interpretive concept recognizes that there can be multiple kinds of conversation happening in any mathematics classroom, each of which assigns fluid roles to the participants. There are passive roles and active roles, just as there are stars and bit parts in dramas. Interactive Positioning occurs when one person positions another; reflexive Positioning occurs when one positions oneself in the conversation. Positioning is not necessarily intentional. (Wagner & Herbel-Eisenmann, 2009, p. 2)

The words that are spoken and written in mathematics classrooms send students messages about who they are as learners, what mathematics is, what it means to know and do mathematics, and so on. Interactions are crucial to identity development and to how students are positioned: “Interactions simultaneously construct how students are positioned as people and as learners, and are a powerful indicator to students about how they are viewed” (Gibbons, 2006, p. 64). If we consider that teachers, textbooks, and students all have some agency within a classroom environment, at different moments in time, each of these participants might be privileged in different ways and may take on responsibilities for the teaching-learning process (Herbel-Eisenmann, 2009).

All participants are involved in the constitution of positioning, and positioning happens at many levels (e.g., in small-group work, whole-class discussions, outside of the classroom in portrayals of mathematics, and at the school level in forms such as tracking students). As the above quotation highlights, people can be positioned by others, which is known as interactive positioning (e.g., “Carl’s smart and he got that answer, so it must be right”), or they may position themselves in different ways, which is called reflexive positioning (e.g., “But his answer is the same as the one I gave earlier”; Davies & Harré, 1999). Positioning is important because it recognizes that authority, agency, and power are dynamic, always changing, and negotiated in interactions between people. Positioning is also constituted by valued artifacts (e.g., textbooks) and how they are used.

Positioning provides a way to think about answers to questions that often go unexamined when focusing only on productive discourse, for example: Who is considered knowledgeable in my classroom? About what (e.g., procedures? concepts?) Whose voice is being heard? In what ways? Who is considered a struggling learner? In our previous work, the secondary mathematics teachers said that they did not consciously consider positioning and expressed that they thought secondary students already knew how to “do school,” so a focus on social aspects seemed relatively unnecessary. (This has been confirmed again in our pilot work.)

In terms of the mathematics, a focus on positioning can help teachers consider epistemological and philosophical questions such as: What does it mean to know mathematics in my classroom? Is mathematics about procedures, concepts and/or something else? Who (e.g., teacher, students) or what (e.g., the textbook) has the authority to determine whether an answer is correct? In what kinds of mathematical practices (e.g., argumentation, explanation, the articulation of only answers) do we engage? What is emphasized, thinking processes or doing processes? Do we generate mathematics collaboratively, or is it something done individually? In the MDISC materials, we have described this focus as occurring at the classroom level because it more holistically describes the kind of practices one engages in when one does mathematics in classrooms.

The dual focus on positioning (between people and what constitutes doing mathematics) is particularly important, given the extensive work done, for example, by scholars such as Boaler and her colleagues (e.g., Boaler, 1998; Boaler & Greeno, 2000; Boaler, Wiliam, & Zevenbergen, 2000). This research points out the importance of students’ experiences in terms of how students come to define what it means to know and do mathematics, as well as whether they decide to identify with mathematics or not. For example, these issues have been shown to affect high-achieving girls (Boaler, 1997), as well as a large group of students who are silently disengaged from these activities (Nardi & Steward, 2003).

Positioning in the MDISC Materials

In the MDISC materials, the idea of positioning is initiated when participants are first asked to solve a mathematical task in the introductory materials. After participants work in small groups and share solutions with the large group, they are asked to reflect on their participation in the group by considering questions such as How did you participate in your group? Whose ideas were taken up as you solved the task? How did you draw on the ways you are “smart” (Featherstone et al., 2011) as you solved the task? What status relationships seemed to surface as your group interacted? These types of prompts appear after every mathematical task as part of the discussion.16 Not...
only do we want participants to consider positioning in the cases, but we also want them to talk about their own experiences of it.

The idea of positioning itself becomes explicit in Constellation 2 as participants consider, more generally, what they think it might be like to participate in the whole-class discussions in the case that is provided. The participants then read a touchstone document about positioning. Later, participants choose one of three students to follow through the case to look at the positioning of a particular student across a lesson. In the remainder of the constellations, participants articulate social goals for their planning and consider how they might support those social goals, compare two different teachers’ interactions with students in small groups in terms of authority issues, and focus on the ways in which students and their solutions are positioned in whole-group discussions.

To consider the positioning of mathematics, participants examine the ways in which tasks are framed, the kinds of activities the students engage in, and the kinds of processes (or verbs) that are apparent. For example, a concrete way to examine what it means to know and do mathematics is to look at the processes or verbs that are highlighted throughout the transcript. Participants can find “doing” verbs and recognize when students are engaging in nonmathematical processes (e.g., take, go up and show, pick) or engaging in mathematical processes (e.g., subtract, solve, count). Participants can also consider whether evidence of “thinking” processes (imagine, figure out, think about, visualize) appears in the discourse. The presence of these processes acknowledges that particular things are going on in students’ minds and that mathematics is also about stopping to engage in reflection. These ideas are continually considered throughout the remainder of the constellations. In the following section, we provide examples of the connections between positioning and TDMs that appear in the MDISC materials.

Positioning and the TDMs

In the MDISC materials, we introduce the TDMs by contrasting them with the more common I-R-E interaction pattern. By contrasting these two types of interaction patterns, it is possible to see how opening up the discourse positions students as needing to be involved in their mathematical learning. When the classroom discourse is opened up, there is also the potential to position students’ ideas as important because space is made for their ideas to be substantively considered.

We also highlight that when the discourse is opened up, participants are able to get more information about how students are making sense of mathematics, but they are also better informed about how students interactively position themselves and others. In fact, rather than watch a video to speculate about how students may experience the classroom interactions, we sometimes assign participants roles as students and the teacher as if they were characters in a play. We include various scenarios that could draw attention to positioning, such as instances of a teacher revoicing but almost completely reformulating the student’s contribution, or naming a solution with a student’s name, or the teacher telling a student that his idea is interesting but that the class is not going to explore it now. In order to consider positioning with respect to the mathematical activity, we contrast how students are asked to participate in the I-R-E example with the interactions that illustrate the use of TDMs.

As originally conceptualized by O’Connor and Michaels (1993), revoicing is a move that provides opportunities for students to participate in particular types of complex thinking by “taking on various roles and stances within recurring social contexts that support . . . intellectual give-and-take and its proto-forms” (p. 64). Thus, examinations of revoicing can allow participants to explore how this TDM ultimately positions students as competent mathematical thinkers. In our prior work with secondary mathematics teachers, however, questions were raised about how students might interpret revoicing (Herbel-Eisenmann, Drake & Cirillo, 2009). These teachers also worried that if they revoiced too often, students would not listen to each other because they could rely on the teacher to revoice. Asking students to revoice can be used as a way to turn over some of the responsibility to students. Exploring these alternative interpretations of positioning is important because students’ interpretations might not be consonant with the teacher’s.

When examining the text around the TDMs, we also have participants investigate whether the classroom discourse involves both doing and thinking processes. Requiring students to engage in both of these types of processes has the potential to position students as doers and thinkers while simultaneously positioning mathematics as a discipline in which actions are coupled with reasons and justifications. Often students are asked to take mathematical actions (e.g., measure) or concrete actions that may not be mathematical (e.g., draw) but may not be provided with space to reflect about these at a more general level. Waiting can provide time for reflection and consideration. Also, probing students’ thinking can be used to position mathematics as something that relies on logical explanations. Considering the TDMs in relationship to the positioning of people and mathematics allows teachers to contemplate students’ developing identities and their developing dispositions about mathematics.
Early Reflections: Considering Participants’ Interactions With the MDISC PD

We have seen pilot participants become more observant about when and how they incorporate TDMs in their teaching. One pilot participant said that her lesson planning changed to incorporate the TDMs strategically, in addition to using them when the opportunity arises:

I didn’t do that [planning for the TDMs] before. I would just sit there and go through a lesson and, if I threw it in, great. Now I’m actually looking when I’m planning: Where are good opportunities where I can throw in these TDMs? (Post-pilot program interview)\(^\text{17}\)

Pilot participants initially expressed surprise at how their students thrived in the interactional spaces that the TDMs created, and they reported high levels of engagement from the students when they incorporated more TDMs into their interactions. They were surprised at the quality of the interactions they observed, especially when involving students who had previously struggled in their classrooms. Not surprisingly, however, the pilot participants continued to express concern about the amount of time it took to have these kinds of discussions.

The pilot participants also found it very useful to have names for aspects of their classroom discourse and suggested that naming these moves allowed them to notice and talk about their classroom discourse in ways they had not been able to previously. This preliminary finding is similar to that of others who have done discourse-related work with teachers (e.g., Staples & Truxaw, 2007; Herbel-Eisenmann & Cirillo, 2009).

In terms of the mathematics register, we have seen pilot participants shift from talking only about whether students use mathematics vocabulary or not to attending to various ways students refer to mathematical ideas and processes. For example, one pilot participant said he now listens carefully to the language students use first and then revoices and integrates students’ ways of communicating with more formal ways of communicating. Some participants expressed that they were less comfortable having students use the more formal discourse required by the mathematics register, however:

I always had a difficulty with the Language Spectrum and the math register because, I mean, obviously we want them to use the correct language. I want them to sit there and say “the parallel lines” instead of saying “these two lines,” or pointing at them. I want them to be able to use the vocab. But I always liked promoting to kids . . . if you can at least get your message across, even if you have to point or something, I will be fine with it. (Post-pilot program interview)

Other pilot participants focused on supporting students’ use of the mathematics register but emphasized that they needed to be more purposeful about the communication contexts they had students work in in order to help students develop more precise language use. Still other pilot participants expressed that they now saw how important it was to build on students’ less formal ways of referring to mathematical objects to support them in developing more precise mathematical discourse.

We have also seen shifts in how pilot participants consider the ways in which classroom discourse might be powerful for student learning by attending to aspects of positioning. They articulated how important it is to carefully consider the social norms in their classrooms, for example, by realizing the importance of being explicit about expectations for participation. Many of the pilot participants noted that serious engagement with students’ ideas contributed to powerful discourse because, as one participant noted, doing so “validated and valued students’ ideas and work—it empowered students in their preparedness to discuss math ideas and in the quality of their discussions” (participant exit card). Yet we admit that there were many situations in which pilot participants talked about student positioning in ways that were not quite what we hoped. For example, there were many instances in which participants would say things such as, “My low-level kids could never do this,” or “I definitely couldn’t have this kind of conversation with my English language learners.” In fact, these kinds of comments motivated us to make substantive changes that highlighted additional aspects of status and smartness before we sent the MDISC materials out to our external pilot sites. We are interested to see how these revisions impact the discussions of positioning in the next round of piloting.\(^\text{18}\)

In terms of positioning of mathematics, pilot participants noted that choosing tasks that tell students what to do...

\(^{17}\) These post-pilot program interviews were conducted by Horizon, Inc., as part of our project evaluation. The findings from the interviews, along with some direct quotations, come from a report shared with us by our evaluators.

\(^{18}\) Additionally, we are currently doing a more thorough analysis of the discursive construction of students in relationship to the activities and questions offered in the MDISC materials in order to better inform our next iteration of revisions.
Rather than leaving them open ended) restricts language associated with hypothesizing and exploring. By doing so, students may come to believe that mathematics does not involve these important practices.

Although we have described the Language Spectrum and positioning in separate sections in order to articulate each in detail, we would be remiss if we did not recognize that these aspects of discourse do not occur in isolation of one another. Some of the pilot participants’ discussions indicate that grappling with how the dual purposes of productive and powerful discourse might support each other and how they might run counter can allow opportunities to wrestle with dilemmas associated with being purposeful about one’s classroom discourse. The early piloting work suggests a need to investigate the ways in which teachers come to understand the TDMs in relationship to productive and powerful discourse as well as how they plan and enact classroom discourse in more purposeful ways.

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APPENDIX A

The following is an overview of the professional development materials.

**Constellation 1: Explanations, Evidence, and Tacit Expectations** focuses primarily on students and the kind of evidence participants might gather in various communication contexts (e.g., writing, small-group interactions, whole-class discussions).

**Constellation 2: Interaction Patterns and Teacher Discourse Moves** focuses primarily on the role that teachers play in shaping classroom discourse. Here we introduce the Teacher Discourse Moves (TDMs), which are specific tools that participants can use to open up the mathematics classroom discourse. We consider how the TDMs can be used to (a) increase the quantity and quality of student talk about mathematics, (b) help participants access their students’ thinking, (c) provide students with opportunities to help other students develop new understandings of mathematics, and (d) empower students as active participants in their mathematical experiences.

**Constellation 3: Planning for Rich Discourse** focuses on planning for powerful and productive classroom discourse. Specifically, the discourse practices introduced in Constellations 1 and 2 are considered within the lesson planning process. Therefore, participants continue to reflect on potential uses for these tools in shaping students’ opportunities to learn mathematics. We also continue to build on participants’ thinking about the role of communication context and consider the ways in which mathematical goals, social goals, and classroom expectations help in planning for rich discourse.

**Constellation 4: Setting Up and Gathering Evidence of Student Work** focuses primarily on the implications for classroom discourse through task launches and independent or small-group work time. In this Constellation, we explore the ways in which a participant’s launch to a mathematical task shapes subsequent discourse. Also, we consider the ways in which a participant’s monitoring during small-group work influences the mathematical activity of those small groups while simultaneously setting the stage for later whole-class discussions.

**Constellation 5: Concluding and Contemplating Evidence** focuses primarily on the conclusion of mathematics lessons and on evaluating student learning. We continue to explore participants’ roles in orchestrating whole-class discussions and how student discourse in that communication context can provide meaningful information about what they do or do not know or understand. Further, we consider how evidence, in its many forms, can be collected at all points in a lesson in order to inform subsequent teaching decisions.